Exercise 2

A vector $\mathbf{v}$ has components $v_x = 1$, $v_y = 2$, $v_z = -5$. A vector $\mathbf{w}$ has components $w_x = 3$, $w_y = -1$, $w_z = 1$. Evaluate:

(a) $(\mathbf{v} \cdot \mathbf{w})$

(b) $[\mathbf{v} \times \mathbf{w}]$

(c) The length of $\mathbf{v}$

(d) $(\boldsymbol{\delta}_x \cdot \mathbf{v})$

(e) $[\boldsymbol{\delta}_x \times \mathbf{w}]$

(f) $\phi_{\mathbf{vw}}$

(g) $[\mathbf{r} \times \mathbf{v}]$, where $\mathbf{r}$ is the position vector.

Solution

(a) $(\mathbf{v} \cdot \mathbf{w}) = v_x w_x + v_y w_y + v_z w_z = 3 - 2 - 5 = -4$

(b) $$[\mathbf{v} \times \mathbf{w}] = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & 2 & -5 \\ 3 & -1 & 1 \end{vmatrix} = (2-5)\hat{\mathbf{x}} - (1+15)\hat{\mathbf{y}} + (-1-6)\hat{\mathbf{z}} = -3\hat{\mathbf{x}} - 16\hat{\mathbf{y}} - 7\hat{\mathbf{z}}$$

(c) $|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{1^2 + 2^2 + (-5)^2} = \sqrt{30}$

(d) $(\boldsymbol{\delta}_x \cdot \mathbf{v}) = \langle 1, 0, 0 \rangle \cdot \langle 1, 2, -5 \rangle = 1$

(e) $$[\boldsymbol{\delta}_x \times \mathbf{w}] = \langle 1, 0, 0 \rangle \times \langle 3, -1, 1 \rangle = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 1 & 0 & 0 \\ 3 & -1 & 1 \end{vmatrix} = -(1-0)\hat{\mathbf{y}} + (-1-0)\hat{\mathbf{z}} = -\hat{\mathbf{y}} - \hat{\mathbf{z}}$$

(f) $$\phi_{\mathbf{vw}} = \cos^{-1}\left(\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}||\mathbf{w}|}\right) = \cos^{-1}\left(\frac{-4}{\sqrt{30}\sqrt{3^2 + (-1)^2 + 1^2}}\right) \approx 102.72^\circ$$

(g) $$[\mathbf{r} \times \mathbf{v}] = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ r_x & r_y & r_z \\ 1 & 2 & -5 \end{vmatrix} = (-5r_y - 2r_z)\hat{\mathbf{x}} - (-5r_x - r_z)\hat{\mathbf{y}} + (2r_x - r_y)\hat{\mathbf{z}}$$